Diagnostic Teaching Lessons

Differentiating means teaching effectively to students with different mathematical backgrounds; it does not mean grouping students according to data...that is a classroom management tactic that has a history of failure when done routinely, although it might be a useful tactic from time to time. This section describes diagnostic teaching and a routine way of using it to manage differences and achieve grade level across these differences.

The premise of diagnostic teaching is to start with what students know and develop it up to grade level. This contrasts sharply with starting with what they don't know, interpreting it as gaps...empty voids...and filling the gaps. Brains don't have voids in them after multiple experiences. Students who have "covered" fractions for years and been "remedied" for fractions and still get fraction problems wrong do not have gaps and voids where fractions ought to be, they have a vast and lively clutter of good ideas, facts, fallacies and bad ideas and most of all, bad memories. Diagnostic teaching begins with who the students really are and what they really bring to the lesson. As messy as the variety of what students bring, the lessons are not messy. They can be stunningly efficient, covering mathematical ground that sweeps over years of curriculum.

There are three parts to this diagnostic teaching lesson section:

1. The structure (classroom management) of the lesson
2. Student engagement and motivation
3. The teaching-learning progression of the lesson

Each section addresses one of the three types of teaching goals every teacher must achieve: managing the classroom, motivating the students and student learning.

1. The structure (classroom management) of the lesson

The prototypical classroom management of diagnostic teaching lesson is given below. Variations play off this basic design. Some variations are by design and some are in response to student responses.

Pose problem	whole class	3-5 min
Start making sense of the problem	solo think	1 min
Solve problem	Pair listen and talk	5 - 10 min
Prepare to present sense made of problem	pair	5 min
Selected Students (3 to 5) present	whole class	15 min
Closure	whole class	5 min

This structure is the basic routine which students can and will get used to. It should not be followed like a script, but used like 'writer's workshop' as a way of working collaboratively. A brief discussion of each part of the lesson follows:

Posing the problem

Whole class: pose problem, make sure students understand the language, no hints at solution. Focus students on the problem situation, not the answer-getting game. Hide the question and ask them to formulate questions that make situation into a word problem. For example, *"A dragonfly flies 50 meters in 2 seconds. ...." . What question could we ask about this situation that uses this information in a word problem? Students might say, How far will it go in 10 minutes? Or How long will it take to fly a kilometer? Or how fast is it flying? Then reveal the actual question, "A dragonfly flies 50 meters in 2 seconds, how long will it take to fly 385 meters?"*

A few minutes focusing on the quantitative situation and the language will help students with language development issues and ELLs. See this as time to teach the language of mathematics problems. This includes the genre conventions of word problems. Teach paying attention to where the quantities are expressed and the relationships between the quantities. Teach them not to dwell on dragonflies. The relevant prior knowledge in this problem is about distance, time and motion; not dragonflies.

It is essential not to give any clues about how to do the problem because this will corrupt the diagnostic value of the lesson. You want to see what the students bring, not if they can follow your directions. You are not interested in what they don't know, but in what mathematics they use to make sense of the problem situation. The varieties of mathematics they use will be the starting point, like it or not, for teaching grade level content.

Solo Thinking: Starting to make sense

Solo honors 'thinking'. When we say "think" we are speaking to an individual. The sound of individuals thinking is silence. It is easy to manage one minute of silence. One minute is manageable for all, 2 minutes will be too long for some and create classroom management issues that aren't worth it. Tell them they will not have time to finish. They are responsible for being able to explain their approach, their way of thinking about the problem to their partner and possibly the class later. They should draw any diagrams or tables or symbols or words that can help them explain their thinking. Silence with pencil in hand, paper on table. You circulate enforcing silence and making happy faces when pencils hit paper.

Pairs make sense and solve problem

In a diagnostic lesson, you want to see the differences so self selected pairs will work well most of the time. Like tend to pair with like. This maximizes the between pair differences. As you interact with the pairs, prompt social norms of listening to each other by asking Alice what Alesia thinks. Model curiosity about and respect for the thinking of others.

As you circulate, try to identify the different ways of thinking in the class. Preserve the differences. Start thinking about who can present each way of thinking. You will only have time for 3 (plus a few minor variations). When you decide to call on a student that might be frightened, do some social scaffolding in advance I like this way you are thinking about this problem. I want the other students to understand it. Make sure you explain this diagram, I want them to understand how it works. Otherwise let all the students prepare for presentation, even though you will call on only a few.

Insist on replacing hand waving with drawing diagrams, writing things down, using labels, using the language of mathematics to express yourself. Alice can't see what you are pointing to when you point to something inside your head. Write something down, draw something that will help Alice see what you are thinking. This is extremely valuable for developing fluency with expressions, diagrams and other representations.

Pairs Prepare to present sense made of problem

The preparation should be a focused team effort by partners. Once the culture of partner work has been established, partners can present to other partners at tables, and then tables can prepare for whole class. But start with partners. Preparation means making a poster or equivalent that can help make the thinking clear to the class during the student presentation. It includes and goes beyond "show your work" . It does not include decoration that is really procrastination, although keep your sense of humor. If it's funny, laugh. I like posters because all can go up on the wall and then the whole class discussion can be about a selection of 3 - 5, and still, everyone sees everyone's work.

Technology has created other options that have other advantages like efficiency (time is precious), memory (it is cool to be able to revisit something produced last week, dynamic geometry and graphing software. White boards work, but only some can fit, so what do the others do while the some are fitting?

Overheads do not work well because you can only see one at time and they go by too fast for many students. They are controlled from the front instead of each student being able to look where their thinking takes them. It is at this micro level that individual pace matters, not the macro level. The macro level is the by-product of the micro level. Overheads create an illusion of controlling the students' attention, just like talking does.

Posters really work for me because all partnerships are equally responsible for preparation working in parallel and students can cast their eye wherever their thinking takes them during closure.

Closure

Whole class listens to and interacts with selected presenters. Presenters present in order selected by the teacher. Start with the way of thinking that is easiest to begin with, easiest to think. This will usually NOT be the easiest way to get the answer and will usually be based on using familiar mathematics rather than grade level mathematics. There is usually good sense making related to the problem situation (high level use of low level mathematics). Example: *it flies 50 meters in 2 secs, 100 in 4, 200 in 8, 300 in 12, 400 in 16...385 in a little less than 16 seconds...15 point something because 375 would be 15. Here's my table.* Then work progressively through ways of thinking ending with the way closest to the chapter's target mathematics. You will have time for 3-5.

The class room dynamic is this: you call on the presenter, they present, you interrupt when they go too fast and say *"wait. Wait. How did you get from B to C? " or "wait, wait. What do the numbers in this column mean? What do they refer to in the problem?"* In other words, you model what someone does who wants to understand the other person's thinking. Especially important when they are going through something that might be confusing. When I am confused, I assume I am not the only one.

Having the posters up simultaneously allows for finding and marking with bright markers differences and correspondences. *Draw a circle around the increase of 5 in y for each increase of 1 in x from a poster of a partner that plugged away 1 by 1 and connect with a drawn sweep to the poster that has a table 'see these are ratios and are all equivalent to 5/1' and then sweep with a line to another poster that has the equation y = 5x, circle the 5 and ask why is this a 5? and a sweeping line to another poster where you draw a circle around the 5 value on the y-axis of a graph over to the value of y corresponding to x=1 and then to the class and tell how smart they are getting.* Sometimes these closing sweeps are magic, but they don't have to be and usually they are not.

2. Student engagement and motivation

Posing the problem
In addition to the language development purpose of these few minutes, it is a chance to motivate some interest in the problem. Asking students to come up with questions is motivating for them. It is a good idea to write the questions down where everyone can see them. It is also a good idea to write down the name of the authors (students) so they get public credit. This has been shown to increase student participation. You chose students to contribute so it isn't the same few all the time and shy students are given their share of credit.

Solo thinking:
An unfinished problem has more mind on it than a solved problem. When a problem is solved, the mind shuts down and moves on. Better to interrupt the individuals when they are just getting interested in the problem, before they are discouraged. At that point, about 1 minute, they will be the most talkative. And they will be more likely to listen. Because the student will have to explain his way of thinking about the problem, he will at worst worry about what will I say? That worry for 1 minute is not bad, it will generate a few ideas at least, enough to keep the process moving along. If you wait 2 minutes, that worry will follow its habitual course to who cares, whatever.

Pairs solve problem
That someone is listening to you and wants to know what you think is an almost irresistible motivation to talk. This especially important for students who shy away from mathematics. As long as the focus of the lesson is 'ways of thinking' about the problem and the mathematics is a way of thinking, the threat level is minimized. If the focus is answer getting, the threat is maximized for many students who aren't sure they did right, or are afraid to try because they might be wrong.

Pairs are the most robust and reliable social organization for this kind of diagnostic lesson. Larger groups (especially more than 3) have hiding places and tend to subside into someone's dominant way of thinking too quickly. The same few students who want and are aggressive about getting attention get a disproportionate share of the eartime. The same many who want to hide are able to hide. The procrastinators can have a field day. Groups of four or more can be deceptively comfortable for students and teachers by hiding issues that should be dealt with. Students who feel, "leave me alone" cannot prosper if they are routinely left alone. Groups of four or more can be a way of leaving them alone. A teacher who interacts with the same sub-group most of the time is not teaching those who need it most. Groups can be another way for the same subset to monopolize interactions. Pairs maximize accountability: no place to hide. Pairs optimize ear time: everyone is listened to.
And pairs teach three basic algebra skills best: how to do homework with someone else, how to study with someone else, and how to tutor and be tutored.

Prepare to present sense made of problem
All students need to be involved in preparation. This is motivated by getting all posters up. Don’t announce who will present until prep time is up. Interact with pairs to get them to make explicit the math thinking you want the class to discuss. If you need a graph and no one is graphing, try to get a pair that seems close to graphing to produce one. You want divergence; diagnostic; make differences Select 3-5 that show the spread in the ways of thinking., the differences in approaches from easy way of thinking about the problem (not the same as easy way to get the answer) to most mathematically mature

Closure
Also praise good habits. "Nice table. Easy for me to understand." "That's a nice way to make sense of the problem." Also highlight high value content, "What's did she just use to do that? Distributive property. 5 point bonus. thank you". Also thank each presenter by name. Interact with presenters, engage whole class in questions
Object and focus is for all to understand thinking of each, including approaches that didn't work; slow presenters down to make thinking explicit draw with marker correspondences across approaches Converge on mathematical target of lesson

3. The teaching-learning progression of the lesson

The diagnostic teaching lesson follows a progression of learning goals: starting with a goal of diagnosis, through making the students' own ways of mathematical thinking understandable to each other, and then pulling all students' thinking toward the way of thinking that is grade level target mathematics.
* Diagnostic: make differences visible; what are the differences in the mathematics that different students bring to the problem. Focus class on the three or four ways of thinking that students bring to the problem.
* All understand each: each other's thinking and the correspondences across ways of thinking: from easiest to understand to most mathematically on grade level.
* Converge toward main grade level mathematics of the chapter: Fit target content into student reasoning. Pull students together through the differences and correspondences in their thinking toward the chapter's mathematical focus.

Each diagnostic teaching lesson teaches the whole unit or chapter. Each lesson "covers" few weeks in few days. Lessons build content by adding mathematical structures and increasing the resolution of mathematical details through the social process of understanding each others ways of thinking as the most efficient route toward the mathematical way of thinking that is the target of the unit or chapter. Work with a variety of problems builds more robust and general understanding and technical know-how. This is not "spiraling," this is depth and thoroughness for durable learning.
Close
Use student presentations to illustrate and explain the key mathematical ideas of lesson
Applaud adaptive problem solving techniques that come up, the dispositional behaviors you value, the success in understanding each others thinking (name the thought)

As a result of the diagnostic lesson, you will know where to spend time in the upcoming unit, where many of the difficulties will be and where to save time. This last, where to save time is as important as where to spend it. A good diagnostic lesson will take students a few weeks into a chapter in just a few days. You will know what student knowledge you are building on.

The partnering strategy for a unit follows a progression from diagnostic goals early to achievement for all late:
• Early: diagnostic, organize to make differences visible. Pair like students to maximize differences between pairs
• Middle: spend time where diagnostic lessons show needs.
• Late: converge on target mathematics. Pair strong with weak students to minimize differences, maximize tutoring